**Topics: Normal distribution, Functions of Random Variables**

1. **The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?**
2. **0.3875**
3. **0.2676**
4. **0.5**
5. **0.6987**

Ans - This means that the car must be ready within 60 minutes (1 hour) - 10 minutes = 50 minutes. The z-score for 50 minutes is (50 - 45) / 8 = 0.625. The probability that a standard normal variable is less than 0.625 is 0.7257. The probability that the service manager cannot meet his commitment is 1 - 0.7257 = 0.2743, or 27.43%.

The correct answer is **B. 0.2676**

1. **The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.**
2. **More employees at the processing center are older than 44 than between 38 and 44.**
3. **A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.**

Ans -

A. False. The mean age is 38 and the standard deviation is 6, so 68% of the employees are between 32 and 44 years old. This means that more employees are between 38 and 44 than older than 44.

B. True. The mean age is 38 and the standard deviation is 6, so 16% of the employees are younger than 32. This means that about 36 employees are under the age of 30.

1. **If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.**

**Ans** - Analyze the two random variables separately and discuss their distributions and parameters:

2X1:

Distribution: Since X1 is normally distributed with mean μ and variance σ^2, multiplying it by 2 results in a new random variable with mean 2μ and variance (2σ)^2. Therefore, 2X1 follows a normal distribution with mean 2μ and variance 4σ^2.

Parameters: The mean of 2X1 is 2μ, and the variance is 4σ^2.

X1 + X2:

Distribution: The sum of two independent normal random variables is also a normal random variable. In this case, X1 and X2 are independent and identically distributed (iid). Therefore, their sum, X1 + X2, follows a normal distribution.

Parameters: When adding two normal random variables, the means simply add up, so the mean of X1 + X2 is 2μ. For the variance, since X1 and X2 are independent, their variances also add up. Thus, the variance of X1 + X2 is 2σ^2.

To summarize:

2X1 follows a normal distribution with mean 2μ and variance 4σ^2.

X1 + X2 follows a normal distribution with mean 2μ and variance 2σ^2.

Both random variables have the same mean, 2μ, but different variances. The variance of 2X1 is twice the variance of X1 + X2. This means that 2X1 tends to have larger fluctuations or variability compared to X1 + X2, given the same values of μ and σ^2.

1. **Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.**
2. **90.5, 105.9**
3. **80.2, 119.8**
4. **22, 78**
5. **48.5, 151.5**
6. **90.1, 109.9**

**Ans –** By using python language library scipy.stats

import scipy.stats as stats

mean = 100

variance = 20\*\*2

# Calculate the standard deviation

std\_dev = variance \*\* 0.5

# Calculate the quantiles for the desired probability range

prob = 0.99

tail\_prob = (1 - prob) / 2 # Probability in each tail

# Calculate the quantiles using the ppf function (inverse CDF)

a = stats.norm.ppf(tail\_prob, loc=mean, scale=std\_dev)

b = stats.norm.ppf(1 - tail\_prob, loc=mean, scale=std\_dev)

# Display the results

print(a)

print(b)

The correct answer is **D. 48.5, 151.5**

1. **Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45**
2. **Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.**
3. **Specify the 5th percentile of profit (in Rupees) for the company**
4. **Which of the two divisions has a larger probability of making a loss in a given year?**

Ans -

Profit1 ~ N(5, 32)

Profit2 ~ N(7, 42)

To convert the profit distributions from dollars to rupees, we multiply the mean and variance by the exchange rate:

Profit1 (in Rupees): ~ N(5 \* 45, 32 \* (45^2)) = N(225, 64800)

Profit2 (in Rupees): ~ N(7 \* 45, 42 \* (45^2)) = N(315, 85050)

To find the Rupee range, we calculate the lower and upper quantiles that contain 95% probability using the cumulative distribution function (CDF) of the normal distribution.

By using python language library scipy.stats

import scipy.stats as stats

**A-Rupee range with 95% probability: [ -9842.001606072465 , 58442.00160607247 ]**

# Calculate the 5th percentile quantile

percentile\_5 = stats.norm.ppf(0.05, loc=mean\_total\_profit, scale=std\_dev\_total\_profit)

# Convert quantile to rupees

percentile\_5\_rupees = percentile\_5 \* 45

# Display the result

print("5th percentile of profit (in Rupees):", percentile\_5\_rupees)

**B- 5th percentile of profit (in Rupees): -4352.870979315509**

To determine which of the two divisions has a larger probability of making a loss in a given year, we compare their mean profit values and see which one is closer to zero.

The mean profit of Division 1 (in rupees) is 225 \* 45 = 10,125 rupees.

The mean profit of Division 2 (in rupees) is 315 \* 45 = 14,175 rupees.

**C- Comparing the mean profit values, Division 1 has a smaller mean profit and is closer to zero. Therefore, Division 1 has a larger probability of making a loss in a given year.**